Examination Period 3: 2018/19

ECN202219N

<table>
<thead>
<tr>
<th>Module Title</th>
<th>Game Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>Five</td>
</tr>
<tr>
<td>Time Allowed</td>
<td>Two hours</td>
</tr>
</tbody>
</table>

Instructions to students:

- Enter your student number **not** your name on all answer books.
- Answer **four** out of **six** questions.
- Begin each question in a separate answer book; label each answer book clearly with the number of the question you are answering.
- The use of a calculator **is** permitted.

<table>
<thead>
<tr>
<th>No. of Pages</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Questions</td>
<td>6</td>
</tr>
</tbody>
</table>
Answer **four** out of **six** questions.

**Question 1**

Mr. Z and Mr. L each own one of two adjacent paddy fields. Such fields rely heavily on rain precipitation and floods for water and nutrients that are essential to the growth of rice. In monsoon seasons, however, rain precipitation becomes too much and it’s best to lead some water away from the fields by a trench. Since water flows freely between Mr. Z’s and Mr. L’s fields, one trench regulates the water level of both fields.

Without a trench, Mr. Z and Mr. L each receive ¥500 (worth of rice) from their own fields. With a trench, each of them receives ¥1000. Digging a trench costs ¥400 (opportunity cost of labour). If just one of them digs, the cost is, of course, born solely by the digger. If they both dig, they share the total cost equally, i.e., ¥200 each.

Mr. Z and Mr. L now decide independently and simultaneously whether to dig or not.

Assume that their preferences are represented by their own monetary payoffs.

a. **Formalize the situation as a strategic game.**  
(9 marks)

b. **Find all the Nash equilibria in pure strategies.**  
(7 marks)

c. **Find the Nash equilibrium in **mixed** strategies.**  
(9 marks)

**Total: 25 marks**
Question 2

Consider the following game between two firms. They have to decide what price to set, either High or Low. Payoffs also depend on the type of Firm B, who can be either “rational” with probability $p$ or “weird” with probability $1-p$. Payoffs are summarized in the following matrices.

Firm B “Rational”

<table>
<thead>
<tr>
<th>Firm A</th>
<th>High Price</th>
<th>Low Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Price</td>
<td>140, 140</td>
<td>20, 160</td>
</tr>
<tr>
<td>Low Price</td>
<td>160, 20</td>
<td>50, 50</td>
</tr>
</tbody>
</table>

Firm B “Weird”

<table>
<thead>
<tr>
<th>Firm A</th>
<th>High Price</th>
<th>Low Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Price</td>
<td>140, 140</td>
<td>20, 120</td>
</tr>
<tr>
<td>Low Price</td>
<td>160, 20</td>
<td>50, 10</td>
</tr>
</tbody>
</table>

a. Write down the entire payoff matrix of the game where B's strategies specify what it will do under each possible type.

(12 marks)

b. Determine the value of $p$ so that the strategy in which Firm B sets Low Price if “Rational” and High Price if “Weird” forms part of a Bayesian Nash equilibrium of this game.

(13 marks)

Total: 25 marks

Question 3

(Voting for a pay rise). Three legislators $(i = 1; 2; 3)$ are voting on whether to give themselves a pay rise. All three want the pay rise; however each faces a small cost in voter resentment $c > 0$. The benefit for the rise is $b > c$. They vote in the order 1-2-3. A pay rise takes place if at least two legislators vote favourably.

a. Draw the game tree that represents this extensive game.

(12 marks)

---

Question 3 continues overleaf
b. What is the outcome obtained by backward induction? 

(13 marks)

Total: 25 marks

Question 4

(Price Competition with Differentiated Products). Consider a market in which two firms produce imperfect substitutes. The market demands for Firm 1’s and Firm 2’s products, $q_1$ and $q_2$, are:

\[ q_1(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2} \]

\[ q_2(p_1, p_2) = \frac{1}{2} + \frac{p_1 - p_2}{2} \]

respectively, where $p_1$ and $p_2$ are the prices set by Firm 1 and Firm 2. For simplicity assume that the marginal cost of production is constant and is set to zero for both firms.

a. Suppose these two firms set their prices simultaneously and independently. What is the Nash Equilibrium outcome? 

(9 marks)

b. Now suppose that the firms choose their prices sequentially. At the first stage, Firm 1 selects its price. At the second stage, after observing Firm 1’s price, Firm 2 chooses its own price.

i. What would be your solution according to Backward Induction? 

(7 marks)

ii. Is there a first mover or second mover advantage? 

(9 marks)

Total 25 marks

Question 5

Show how the phenomenon of overfishing can be represented as a Prisoners’ Dilemma. (hint: set up the game with two players, each of whom can undertake low or high fishing activity). 

(25 marks)
Question 6

Two players, A and B play the following game. First A must choose IN or OUT. If A chooses OUT the game ends, and the payoffs are A gets 2, and B gets 0. If A chooses IN then B observes this and must then choose in or out. If B chooses out the game ends, and the payoffs are B gets 2, and A gets 0. If A chooses IN and B chooses in then they play the following simultaneous move game:

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Up</td>
<td>3; 1</td>
<td>0; 2</td>
</tr>
<tr>
<td>A Down</td>
<td>1; 2</td>
<td>1; 3</td>
</tr>
</tbody>
</table>

a. Draw the tree that represents this game.  

b. Find all the pure-strategy SPE of the game.  

Total: 25 marks