Examination Period 3: 2018/19

Module Title: Data and Mathematics for Economists
Level: Four
Time Allowed: Two hours

Instructions to students:
- Enter your student number not your name on all answer books.
- Answer four questions: two from Section A and two from Section B.
- All questions are equally weighted.
- The use of calculators is permitted.
- Show all your workings clearly.

<table>
<thead>
<tr>
<th>No. of Pages</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
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<td>6</td>
</tr>
</tbody>
</table>
Section A

Answer two out of three questions.

1. a. Differentiate the following functions with respect to x (i.e. find $\frac{dy}{dx}$ and do not simplify).
   
   i. $y = 5x^3 + 0.6x^2 - 5x + \frac{10}{x}$
   
   ii. $y = \frac{x^3 - 5x - 4}{2x + 9}$
   
   iii. $y = (x^4 - 5x^2)^6$
   
   iv. $y = (3x^{-3} + 5x^2 - 2x + 5)(x^2 - 6x + 7)$
   
   v. $y = 6x^2 - 8x^5 + 2x - \frac{3}{4}$

   (15 marks)

b. Sketch (graph paper not required) the graph of the following relationship using the procedures for finding relative maxima and minima:

   $y = 35 + 2x - 3x^2 + \frac{x^3}{3}$

   (10 marks)

Total: 25 marks

2. a. A consumer has the utility function, $U = 85X^{0.5} + 15Y^{0.5}$. The prices of X and Y are £8 and £1 respectively and the consumer has a money income of £100.

   i. How much of X and Y will she consume to maximise utility?

   ii. Calculate and interpret the Lagrange multiplier.

   (15 marks)

b. Write the following simultaneous equation system in matrix form, and solve it using matrix methods:

   
   $\begin{align*}
   2x + 4y &= 2 \\
   -3x + y &= 11 \\
   \end{align*}$

   (10 marks)

Total: 25 marks
3.  

a.  Consider the following average revenue and total cost functions:

\[ P = 450 - Q \]  \ (Note: AR = P)

\[ TC = 0.04Q^3 - Q^2 - 260 + 10,000 \]

where total cost (TC) is measured in £, price (P) is measured in £ per unit and Q is quantity (units sold).

i.  Determine the profit maximising price and quantity.  \( (12 \text{ marks}) \)

ii. Calculate the total revenue, total cost and the maximum profit.  \( (5 \text{ marks}) \)

b.  Find the values of x which solve the quadratic equation:

i.  \( 5x^2 + 6x + 1 = 0 \)

ii. \( -4x^2 - 7x + 12 = 0 \)  \( (8 \text{ marks}) \)

Total: 25 marks
Section B

Answer **two** out of **three** questions.

4.  
   **a.** A data set contains the following ten values:
     
     4 7 2 5 9 3 8 1 0 9

     i.  Find the mean  
         (3 marks)

     ii. Find the mode  
         (2 marks)

     iii. Find the median  
         (4 marks)

     iv. Find the sample variance  
         (6 marks)

     v.  Find the sample standard deviation  
         (5 marks)

   **b.** What are the measures of variability? Explain any **two** measures.  
         (5 marks)

**Total: 25 marks**
5.  
   a. Suppose you work for an insurance company, and you sell a £9,000 one-year term insurance policy at an annual premium of £400. Actuarial tables show that the probability of death during the next year of a person of your customer’s age, gender, health etc. is 0.002. What is the expected gain (amount of money made by the company) for a policy of this type?  
      (10 marks)
   
   b. The following table shows the number of weekly accidents at a factory:

<table>
<thead>
<tr>
<th>No of Accidents per week (x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of weeks (f)</td>
<td>11</td>
<td>17</td>
<td>14</td>
<td>7</td>
<td>1</td>
<td>50</td>
</tr>
</tbody>
</table>

Estimate the probability and the expected value of the accidents at the factory.  

(15 marks)

Total: 25 marks

6. A random sample of 80 observations from a normally distributed population possesses a sample mean equal to 25.3 and a sample standard deviation equal to 3.5.

   a. Find an approximate 95% confidence interval for $\mu$.  
      (15 marks)
   
   b. Define what is point estimate, confidence interval estimate and level of confidence.  
      (10 marks)

Total: 25 marks