Examination Period 3: 2016/17

ECN101317N

Module Title: Mathematics for Economists
Level: Four
Time Allowed: Two hours

Instructions to students:
- Enter your student number not your name on all answer books.
- Answer three questions: two from Section A and one from Section B.
- The use of a non-programmable calculator is permitted.

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No. of Questions | 5
Section A

Answer two out of three questions

1. a. Derive the Linear equation which passes through the following (x,y) coordinate pairs, (3,5) and (7,3). Sketch its graph. (10 marks)

b. Find the values of x which solve the quadratic equation:

i. $3x^2 - 7x - 20 = 0$

ii. $3x^2 + 13x - 10 = 0$ (8 marks)

c. A firm’s Average Revenue (AR) relationship is represented by $P = 438 - Q$, which is also the inverse demand curve, where P is price and Q is the quantity produced and sold. Work out the firm’s Total Revenue (TR) and Marginal Revenue (MR) relationships from this information. The firm’s Total Costs (TC) are given by $TC = 18Q + Q^2$. Sketch the marginal cost and marginal revenue relationships and calculate the output at which profit is at a maximum. How do you know that you have found the profit maximising output? (12 marks)

(Total: 30 marks)

2. The wholesale market supply and demand curves for a brand of tennis rackets are estimated to be $Q_s = -30 + 4.5P$ and $Q_d = 300 - 6.5P$ respectively. $Q_s$ is the quantity supplied, $Q_d$ is the quantity demanded and P is price in £ per racket.

a. What is the equilibrium price and quantity for this brand of tennis rackets? (15 marks)

b. Work out the price elasticities of supply and demand for $P = 26$ and interpret your answers. (15 marks)

(Total: 30 marks)

Section A continues overleaf
3.  

a. Differentiate the following functions with respect to \( x \)  
(i.e. find \( \frac{dy}{dx} \) and do not simplify).  

i.  \( y = -3x^4 - 5x^3 - 2x^{-1} + 14 \)  

ii.  \( y = \left( -2x^3 + 3x^{-4} \right)^5 \)  

iii.  \( y = \frac{\left(3x^2 - 5x + 2\right)}{x^3 + 4x} \)  

(10 marks)  

b. Sketch accurately (graph paper not required) the graph of the following relationship using the procedures for finding relative maxima and minima:  

\[ y = \frac{x^3}{3} + 2x^2 - 45x + 25 \]  

(10 marks)  

c. Solve the following simultaneous equation system:  

i.  \(-x - 3y = -1 \) and \( 3x + 5y = -5 \)  

ii.  \(5x - 2y = 17 \) and \( 2x - 4y = 10 \)  

(10 marks)  

(Total: 30 marks)
Section B

Answer one out of two questions.

4.

a. Consider the production function given by \( Q = 10K^{\frac{1}{2}}L^{\frac{1}{2}} \), where \( Q \) is the output rate, \( K \) is the input of machine hours, and \( L \) is the input of labour hours. Work out the total differential of the output rate.

(10 marks)

b. The demand for a product is given by \( Q = -\frac{5P^2}{4} + \frac{Y^2}{3} + 1500 \), where \( Q \) is the quantity demanded, \( P \) is the price of the product, and \( Y \) is income. Calculate the partial price elasticity of demand at \( P = 26 \), \( Y = 45 \) and interpret your answer.

(15 marks)

c. Consider the utility function, \( U = 16X_1^{\frac{3}{4}}X_2^{\frac{1}{4}} \) where \( X_1 \) is the consumption of commodity 1, \( X_2 \) is the consumption of commodity 2, and \( U \) is total utility.

i. Show that the elasticity of utility with respect to the consumption of commodity 1 is equal to \( \frac{3}{4} \).

ii. Calculate and interpret the value of the Marginal Rate of Substitution (MRS) at \( X_1 = 150 \) and \( X_2 = 40 \)

(15 marks)

(Total: 40 marks)
5.

a. A firm has a production function given by \( Q = 6K^{\frac{1}{3}} + 9L^{\frac{1}{3}} \), where \( Q \) is the output rate, \( K \) is the number of machine hours, and \( L \) is the number of labour hours. The iso-cost relationship is given by \( 2K + 3L = 100 \).
   
   i. Work out the amount of machine hours and labour hours that are needed to maximise output, \( Q \).
   
   ii. Calculate the Lagrange multiplier and interpret its value.

   (15 marks)

b. An individual has a utility function, \( U = 12X_1^{\frac{1}{4}}X_2^{\frac{3}{4}} \) where \( X_1 \) is the consumption of commodity 1, \( X_2 \) is the consumption of commodity 2, and \( U \) is total utility. The price of commodity 1 is 4 and the price of commodity 2 is 12.
   
   i. How much of \( X_1 \) and \( X_2 \) should the individual consume to minimise spending in order to achieve a total utility of 120 units?
   
   ii. Calculate and interpret the Lagrange multiplier.

   (15 marks)

c. Write the following simultaneous equation system in matrix form, and solve it using matrix methods:

\[
-4x + 3y = -27 \quad \text{and} \quad 2x - 5y = 31
\]

(10 marks)

(Total: 40 marks)