Summer Examinations 2015

ENG105115N

Module Title: Mathematics for Engineers
Level: Four
Time Allowed: Two hours

Instructions to students:
- Enter your student number not your name on all answer books.
- Answer all questions.
- Where a question has more than one part the division of marks is stated.
- Begin each answer on a separate page; label each page clearly with the number of the question you are answering.
- The use of a non-programmable calculator is permitted.
- Graph paper and a formula booklet are provided.
- Students are permitted to remove this examination paper at the end of the examination.

<table>
<thead>
<tr>
<th>No. of Pages</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Questions</td>
<td>5</td>
</tr>
</tbody>
</table>

Page 1 of 3
Answer all questions.

**Question 1**

a. By dividing the interval $0 \leq x \leq 1$ into ten strips, use Simpson’s Rule to demonstrate that:

$$4 \int_{0}^{1} \frac{1}{1+x^2} dx = \pi.$$  

(12 marks)

b. Working to 5 decimal places on your calculator estimate the error in calculation by Simpson’s Rule.

(3 marks)

Total: 15 marks

**Question 2**

Evaluate the following integral by the technique of integration by parts:

$$\int e^{-3x} \cos 2x \, dx$$

(25 marks)

**Question 3**

Find the general solution to the following differential equation using the Integrating Factor.

$$\frac{dy}{dx} - \frac{1}{x^2} y = x^2$$

(15 marks)

**Question 4**

Solve the differential equation $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 2e^{-2x}$ given that when $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$.

(25 marks)
Question 5

Given that \( x(0) = x'(0) = 0 \) write the differential equation \( 4x''(t) - 9x(t) = -18 \) into its Laplace transform form and hence formulate a solution to the differential equation.

(25 marks)